Short Papers

Transient Analysis of Partially Coupled Lines

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Abstract—**We propose a time-domain analysis method for partially coupled lines by dividing them into "tubes." A tube of order N is a set of N coupled lines of equal lengths for which transient analysis based on modal decomposition is well known. An interface problem occurs at the junction of tubes of different orders. Such a case is analyzed by introducing two operators, called dilatation and shrinkage.**

I. Introduction

In very large scale high-speed circuits, coupling effects between interconnecting lines play an important role. They may involve distortion on driven signals and introduce spurious ones in nonactivated lines [1]. Therefore, the transient behavior of coupled interconnections must be known accurately before designing a performing circuit.

Transient analysis for a set of N coupled lines of equal lengths has been recently reported [3]. In our paper, such a set is referred to as a tube of order N. But, a "tube" itself is an ideal pattern and cannot be representative of coupled lines that exist in physical devices. We propose here a time-domain analysis of coupled lines with discontinuities and of different lengths. The method consists in dividing an interconnection system into several tubes [4]. The discontinuities appear at the interface between tubes of different orders. Such interfaces are analyzed by the method discussed in the following sections.

II. Principle of the Method

As an example, the structure sketched in Fig. 1 possesses two interfaces (1 and 2). Examining this structure, on each side of the interfaces the number of lines may be different. The structure given in Fig. 1 is first decomposed into "tubes." Note that interface 1 of this structure is at the junction of four tubes: T_1, T_2, T_3, and T_4. In a similar way, interface 2 is placed at the junction of three tubes: T_4, T_5, and T_6.

The time-domain behavior of a tube is readily determined by means of a previous modal analysis [5], which provides the decomposition of the coupled signals propagating on each line into decoupled modes. This method assumes a set of uniformly coupled lines along identical lengths. For each mode, a propagating velocity V_j and a characteristic impedance Z_j can be defined and analyzed by the method of characteristics [6]. Then, an equivalent circuit is defined for a tube (Fig. 2).

Note that all the voltages and currents are time dependent. To simplify the representations, the time variable (t) is omitted in the following notations (e.g., $Ua\ (t) \rightarrow Ua$).

The amplitudes of the independent modes are determined by the load and source matrices $[Ra]$, $[Rb]$ and $[Ua]$, $[Ub]$. The tube itself is characterized by its matrix of equivalent impedance $[R]$

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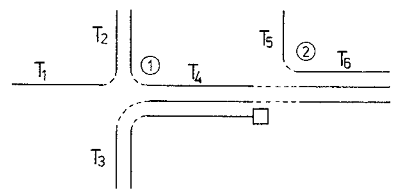

Fig. 1. An example of discontinuous coupled lines

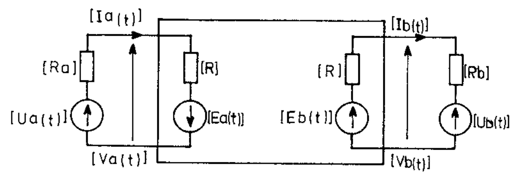

Fig. 2. Equivalent circuit of a tube.

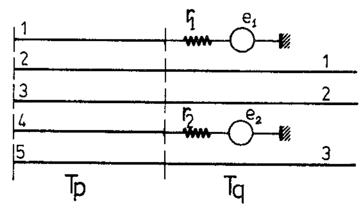

Fig. 3. An interface between two tubes of different orders ($np > nq$)

and equivalent voltage sources $[Ea]$ and $[Eb]$. Only junctions between two tubes are discussed in this paper. Obviously, at the junctions of tubes of different orders, $[Ra]$ and $[Ua]$, designating, respectively, the equivalent impedance and source matrices of the tube, are not of the same order as $[R]$ and $[Ea]$. Therefore, in order to match the different orders of these matrices, "dilatation" and "shrinkage" operators are defined. Depending on the direction of propagation, one of these two operators will be used. Thus, for tubes Tp and Tq such that $Np > Nq$, the dilatation operator will be used in order to analyze the signals driven from Tp to Tq, and the shrinkage one for signals propagating from Tq to Tp.

A. Operators of Dilatation

Consider the tubes P and Q given in Fig. 3 with $np > nq$. Here, the lines 2, 3, and 5 of Tp are continuous, and lines 1 and 4 are discontinuous and loaded by lumped sources and resistances (e.g., e_1 and r_1).

We define a continuity parameter kc as follows:

$$kc(i) = j \tag{1}$$

where j is the order number for continuous lines of Tp, and i is the number of the corresponding line of Tq. For the interface in Fig. 3,

$$kc(1) = 2 \quad kc(2) = 3 \quad kc(3) = 5. \tag{2}$$

In the same way, we define a discontinuity parameter kd:

$$kd(i) = j \tag{3}$$

with j being the order number for discontinuous line of Tp and i the number of the terminating lumped loads.

As a result, in the present example,

$$kd(1) = 1 \quad kd(2) = 4. \tag{4}$$

The parameters kc and kd will be used in the dilatation and shrinkage operators.

Load conditions for Tp are expressed as

$$[Vb]_p = [Rb]_p[ib]_p + [Ub]_p. \tag{5}$$

$[Vb]_p$ and $[ib]_p$ are, respectively, the voltages and currents at the output of Tp. One can use the following equations to determine $[Rb]_p$ and $[Ub]_p$:

$$Vb_1 = r_1 ib_1 + e_1 \tag{6}$$

$$Vb_4 = r_2 ib_4 + e_2 \tag{7}$$

$$\begin{bmatrix} Vb_2 \\ Vb_3 \\ Vb_5 \end{bmatrix} = [R]_q \begin{bmatrix} ib_2 \\ ib_3 \\ ib_5 \end{bmatrix} + [Ea]_q. \tag{8}$$

Using relations (5)–(8), the matrices $[Rb]_p$ and $[Ub]_p$ are evaluated:

$$[Rb]_p = \begin{bmatrix} r_1 & 0 & 0 & 0 & 0 \\ 0 & R_{11} & R_{12} & 0 & R_{13} \\ 0 & R_{21} & R_{22} & 0 & R_{23} \\ 0 & 0 & 0 & r_2 & 0 \\ 0 & R_{31} & R_{32} & 0 & R_{33} \end{bmatrix} \tag{9}$$

$$[Ub]_p = \begin{bmatrix} e_1 \\ Ea_1 \\ Ea_2 \\ e_2 \\ Ea_3 \end{bmatrix}. \tag{10}$$

In these expressions, R_{ij} are the components of matrix $[R]_q$, and Ea_i are the components of matrix $[Ea]_q$. The general expressions of matrices $[Rb]_p$ and $[Ub]_p$ are defined by

$$[Rb]_p = \overset{np}{\underset{nq}{\mathscr{D}r}}[[R]_q] \leftrightarrow Rb[kc(i), kc(j)] = Rij$$

$$\text{for } i, j \leqslant n_q$$

$$Rb[kd(i), kd(i)] = ri$$

$$\text{for } i \leqslant (n_p - n_q) = n_\Delta$$

$$Rb[kc(i), kd(j)] = 0$$

$$\text{for } i \leqslant n_q, j \leqslant n_\Delta$$

$$Rb[kd(i), kc(j)] = 0$$

$$\text{for } i \leqslant n_\Delta, j \leqslant n_q \tag{11}$$

$$[Ub]_p = \overset{np}{\underset{nq}{\mathscr{D}e}}[[Ea]_q] \leftrightarrow Ub[kc(i)] = Ea_i$$

$$\text{for } i \leqslant n$$

$$Ub[kd(i)] = e_i$$

$$\text{for } i \leqslant n_\Delta. \tag{12}$$

Relations (11) and (12) entirely define the operators of dilatation ($\mathscr{D}r$ and $\mathscr{D}e$).

B. Operators of Shrinkage

In Fig. 3, the voltages and currents at the input of Tq are related by the following relations:

$$[Va]_q = [Ua]_q - [Ra]_q[ia]_q. \tag{13}$$

In order to determine the matrices $[Ra]_q$ and $[Ua]_q$, we use the following relations:

$$Vb_1 = r_1 ib_1 + e_1 \tag{14}$$

$$Vb_4 = r_2 ib_4 + e_2 \tag{15}$$

$$[Vb]_p = [Eb]_p - [R]_p[ib]_p. \tag{16}$$

By substituting Vb_1 and Vb_4 from (14) and (15) into (16) and eliminating ib_1 and ib_4, one obtains three equations instead of the five given by (16). On the other hand, lines 2, 3, and 5 of Tp are continuous; therefore,

$$Vb_2 = Va_1 \quad ib_2 = ia_1 \tag{17}$$

$$Vb_3 = Va_2 \quad ib_3 = ia_2 \tag{18}$$

$$Vb_5 = Va_3 \quad ib_5 = ia_3. \tag{19}$$

We finally find the load conditions with the following values for $[Ra]_q$ and $[Ua]_q$:

$$Ra_{ij} = R_{mn} - \frac{R_{1n}R_{m4}R_{41} + R_{4n}R_{m1}R_{14} - (R_{11} + r_1)R_{m4}R_{4n} - (R_{44} + r_2)R_{m1}R_{1n}}{(R_{44} + r_2)(R_{11} + r_1) + R_{14}R_{41}} \tag{20}$$

$$Ua_i = Eb_m - \frac{[R_{m1}(R_{44} + r_2) - R_{m4}R_{41}](Eb_1 - e_1) + [R_{m4}(R_{11} + r_1) - R_{m1}R_{14}](Eb_4 - e_2)}{(R_{44} + r_2)(R_{11} + r_1) + R_{14}R_{41}} \tag{21}$$

$$m = kc(i)$$

$$n = kc(j)$$

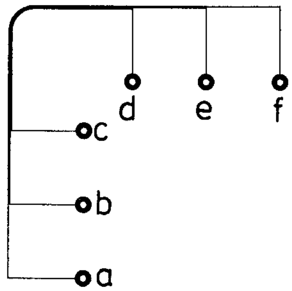

Fig. 4. An example of partially coupled lines.

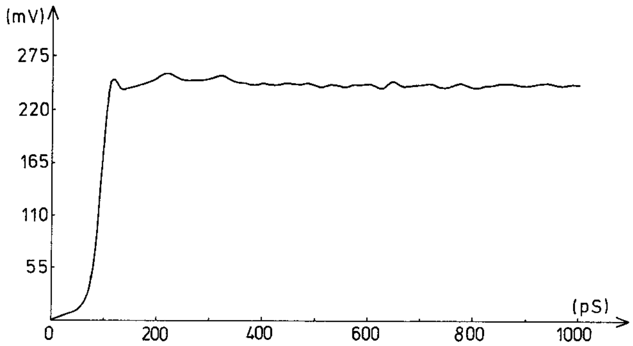

Fig. 5. Signal delivered by the generator.

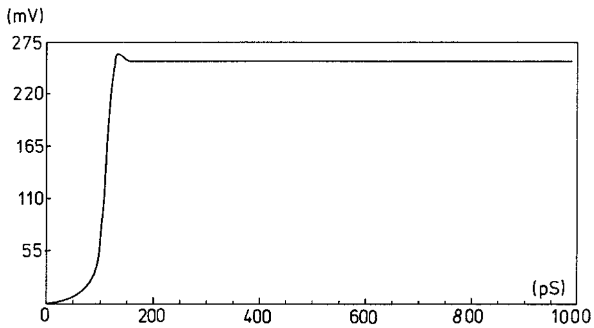

Fig. 6. Approximated shape of the input signal.

In (20) and (21), R_{ij} is a component of matrix $[R]_p$ and Eb_i is a component of matrix $[Eb]_p$.

In order to give these expressions a general form, the following matrices have to be defined:

a continuity matrix $[A]$, which is defined as

$$A_{ij} = R[kc(i), kc(j)], \qquad \text{with } i, j \leqslant nq \tag{22}$$

a discontinuity matrix $[B]$:

$$B_{ij} = R[kd(i), kd(j)], \qquad \text{with } i, j \leqslant n_\Delta = n_p - n_q \text{ and } i \neq j \tag{23}$$

$$B_{ii} = R[kd(i), kd(i)] + r_i, \qquad \text{with } i \leqslant n_\Delta \tag{24}$$

a complementary matrix of the first kind $[C]$:

$$C_{ij} = R[kc(i), kd(j)], \qquad \text{with } i \leqslant nq, j \leqslant n_\Delta \tag{25}$$

a complementary matrix of the second kind $[D]$:

$$D_{ij} = R[kd(i), kc(j)], \qquad \text{with } i \leqslant n_\Delta, j \leqslant nq. \tag{26}$$

In the definitions of A, B, C, and D, the parameters R_{ij} are components of the impedance matrix of T_p ($[R]_p$).

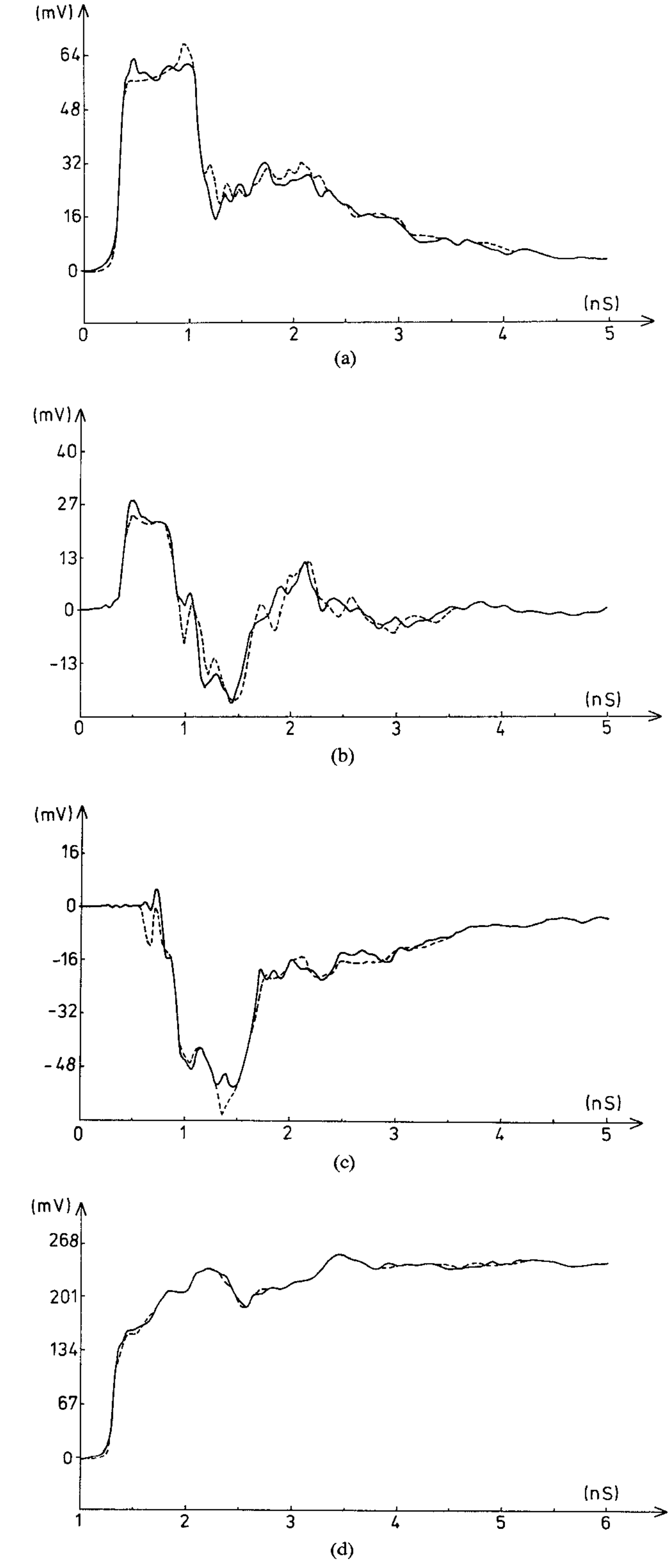

Fig. 7. Results obtained for the structure given in Fig. 4. (a) Signal at point b (b) Signal at point c. (c) Signal at point d. (d) Signal at point f.

In a general form, $[Ra]_q$ and $[Eb]_q$ are written as follows:

$$[Ra]_q = [A] - [C][B]^{-1}[D] \tag{27}$$

$$[Ea]_q = [Eb]_c - [C][B]^{-1}[Eb]_d. \tag{28}$$

$[Eb]_c$ is defined by

$$Eb_i = Eb[kc(i)], \qquad \text{with } i \leqslant nq \tag{29}$$

and $[Eb]_d$ by

$$Eb_i = Eb[kd(i)] - e_i, \qquad \text{with } i \leqslant n_\Delta. \tag{30}$$

$Eb[kc(i)]$ and $Eb[kd(i)]$ are components of matrix $[Eb]_p$.

III. Results

A simulation program has been developed using the operators of dilatation and shrinkage defined in Section II. Different configurations and particularly the structure given in Fig. 4 were tested. The lines are microstrip 0.2 mm wide, and the spacing between the lines in the coupling region is $s = 0.1$ mm. The dielectric substrate has a height $h = 1.55$ mm, a strip thickness $t = 35\ \mu$m, and a dielectric constant $\epsilon_r = 3.0$.

The circuit is divided into tubes of one, two, and three lines with the equivalent impedance matrices given below [5].

For a tube of one line:

$$[R]_1 = [162]\ \Omega$$

for a tube of two lines:

$$[R]_2 = \begin{bmatrix} 159 & 97 \\ 97 & 159 \end{bmatrix} \Omega$$

for a tube of three lines:

$$[R]_3 = \begin{bmatrix} 158 & 95 & 69 \\ 95 & 155 & 95 \\ 69 & 95 & 158 \end{bmatrix} \Omega.$$

The circuit is fed through point (a) by a generator delivering the signal represented in Fig. 5. While simulating the circuit, the input signal is approximated by the curve given in Fig. 6. Point (d) is open ($Zd = \infty$) and the other points are terminated by 50-Ω loads ($Zb = Zc = Ze = Zf = 50\ \Omega$). The output signals are filtered by the oscilloscope used in the measurements. Therefore, we have smoothed the theoretical results by a theoretical filter with $RC =$ 50 ps.

The simulation results (dashed curves) and those of the measurements (continuous curves) are given in Fig. 7. The excellent agreement between the experimental and theoretical results proves the validity of the method.

IV. Conclusions

In a given integrated or printed circuit, some lines may be coupled along some part of their length, and are termed partially coupled lines. The method proposed in this paper provides a straightforward time-domain analysis of this kind of line. It is based on the concepts derived from analysis of continuous lines, and readily applies to any number of nonuniformly coupled lines. Dividing a structure of partially coupled lines into tubes allows one to use transient analysis methods described for continuous coupled lines (modal analysis, characteristics method).

In order to solve the problem of interfaces between tubes of different order, we have introduced two operators, called dilatation and shrinkage. By testing different partially coupled lines, close agreement with theoretical results is obtained, proving the validity of the proposed method. The number of lines is not a limiting factor, and the method can be applied to nonuniformly coupled lines [7]. In this case, the nonuniformly coupled lines can be divided into cascaded tubes with different propagation parameters.

Some authors use methods based on a multiport conception and the scattering matrix to solve the junction problem [8]. The dilatation and shrinkage operators can also be used in these cases when the lines are discontinuous.

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FM Noise in Multiple-Device Oscillators

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Abstract —**The FM noise in a multiple-device oscillator is analyzed. It is shown that FM noise depends on the circuit parameters and the number N of the constituent devices. For a circuit-dependent critical value of N, FM noise is maximum and it is proportional to $N^{-\frac{1}{2}}$ when N is very large.**

I. Introduction

Kurokawa's analysis [1] shows the FM noise in a multiple-device oscillator to be inversely proportional to the external Q and the number of active devices which constitute the oscillator. The analysis, however, assumes external Q to be independent of the number of active devices. More recently, it has been observed that the external Q of a multiple-device oscillator decreases as the active devices are increased in number [2]–[4]. In view of these observations, this paper analyzes the circuit dependence of FM noise in a multiple-device oscillator.

II. RMS Frequency Deviation

Typically, a multiple-device oscillator [1] consists of a number N of identical negative-conductance devices, each provided with a stabilizing conductance G_0 and equally coupled to a power-combining cavity. The cavity is equivalent to a parallel combination of a loss conductance G_c that includes circuit losses other than those in the G_0's, a capacitance C_c, an inductance, and an equivalent load conductance $n_o^2 G_L$, where G_L is the load conduc-

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